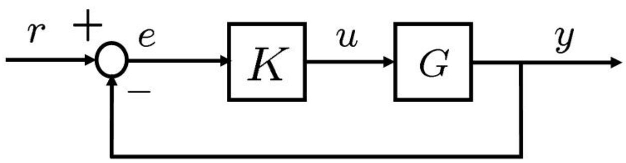
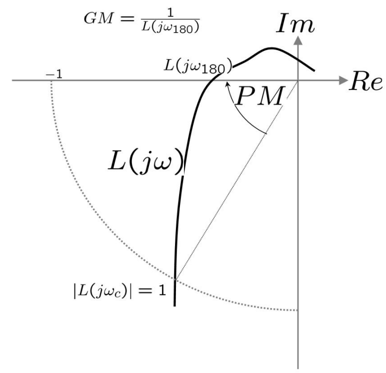
Q1 [20pts]

This question studies some of the fundamental limitations imposed on the control design due to presence of right half zeros in the plant transfer function. Consider the feedback configuration shown below:



(A)



(B)

(a) We use the notation $\|M\|_\infty$ to denote the peak value of a stable transfer function $M(s)$ where

$$\|M\|_\infty = \max_\omega |M(j\omega)| = \max_{Real(s)\geq 0} |M(s)|. \tag{1}$$

Assume that the result $\max_\omega |M(j\omega)| = \max_{Real(s)\geq 0} |M(s)|$ hold true for stable transfer function $M(s)$.

i. Find the transfer function $S(s)$ from r to e (called the sensitivity transfer fuction) from Figure (A). Show that $|S(j\omega)|$ is the reciprocal of the distance of the point $L(j\omega) = G(j\omega)K(j\omega)$ from the point $-1 + j0$ on the Nyquist plot in Figure (B). Deduce that the control design that achieves lower $\|S\|_\infty$ value achieves better relative stability (more robust) for the closed loop system. [3pts]

ii. Assume that the plant $G(s)$ has a right half plane zero at z.

A. Show that $S(z) = 1$. Use equation (1) to show that it is *is not* possible to design a stabilizing controller $K(s)$ such that $|S(j\omega)| < 1$ for all ω (use that fact that $\max_\omega |M(j\omega)| = \max_{Real(s)\geq 0} |M(s)|$ if M is stable). [2pts]

B. Show that it is *not* possible to design a stabilizing controller $K(s)$ such that $\|w_p(s)S(s)\|_\infty < |w_p(z)|$ where $w_p(s)$ is any stable transfer function. [2pts]

(b) Let the plant transfer function be given by

$$G(s) = \frac{3(1-2s)}{(5s+1)(10s+1)}.$$

i. Find the poles and zeros of $G(s)$. Is the plant stable? What would be your objection to using an open loop control strategy $U(s) = G^{-1}(s)R(s)$ for perfect tracking so that $Y(s) = G(s)u(s) = G(s)G^{-1}(s)R(s) = R(s)$? [2pts]

ii. Consider a proportional control design where $K(s)$ is equal to a constant k_c.

A. Find the sensitivity transfer function from r to e for this plant and show that higher values of k_c implies smaller tracking errors. [2pts]

B. Is the system stable for all positive values of k_c? Determine the range of values of k_c (given that $k_c > 0$) that guarantees stability. [3pts]

iii. Find the gain $k_c^\star > 0$, the critical value at which the closed loop system becomes marginally stable (that is, when the closed loop poles are of the form $\pm j\omega^\star$). [2pts]

A. Find the corresponding frequency $\omega^\star$ and find the proportional-integral (PI) controller from the Ziegler and Nichols tuning rules given by

$$K(s) = \frac{k_c^\star}{2.2}\left(1 + \frac{0.6\omega^\star}{\pi s}\right).$$

[2pts]

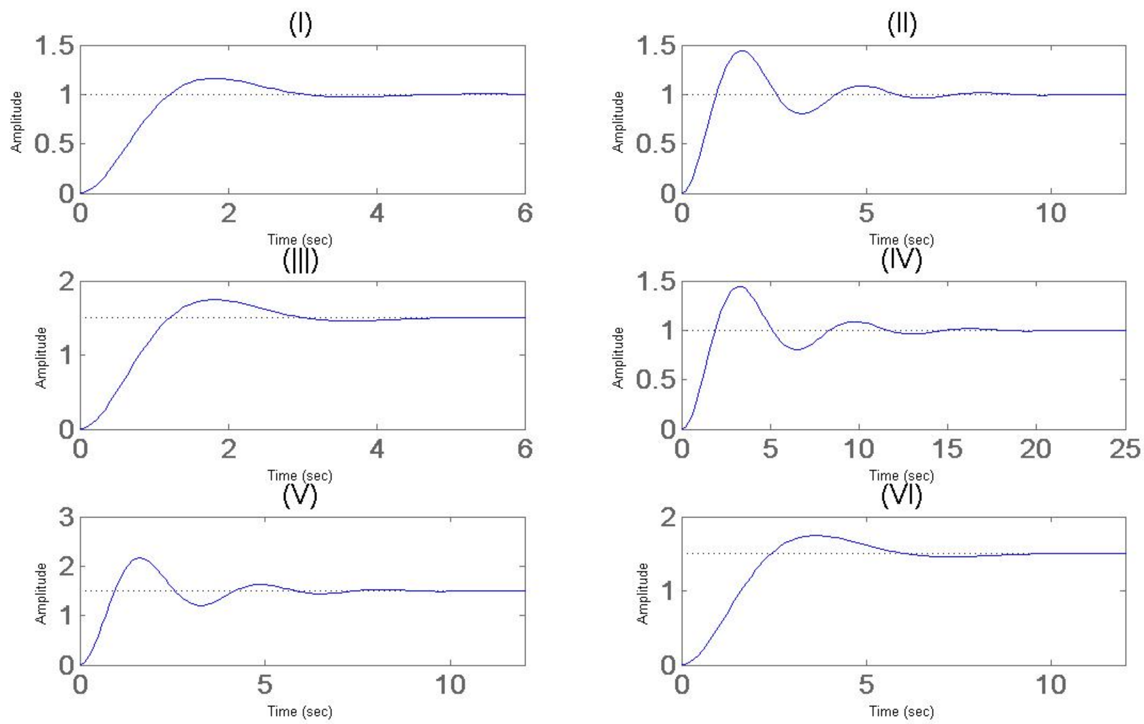
B. Find the steady state error with this control design when the reference signal r is (i) a unit step and (ii) a unit ramp signal. [2pts]

Q2 [15pts]

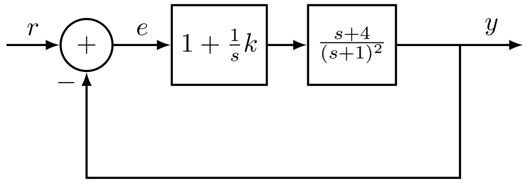
Consider the systems with the following transfer functions:

$$G_A(s) = \frac{1}{s^2+0.5s+1}$$
$$G_B(s) = \frac{4}{s^2+1s+4}$$
$$G_C(s) = \frac{3}{2s^2+2s+2}$$
$$G_D(s) = \frac{6}{s^2+2s+4}$$
$$G_E(s) = \frac{4}{s^2+2s+4}$$
$$G_F(s) = \frac{12}{2s^2+2s+8}$$

You are required to match these with the unit step responses shown below (*Hint: calculate the damping* ζ*, the natural frequency* ω_n *for each system and the corresponding steady state output values*).



Q3 [10pts]



Consider the feedback system shown in the figure, with transfer function $G(s) = \frac{s+4}{(s+1)^2}$ and controller of the form $C(s) = 1 + \frac{1}{s}k$.

(a) Find the range of k values that make the feedback system stable. [3pts]

(b) Can you find a k that achieves a steady-state error of 0.01 for a ramp input, $\frac{1}{s^2}$? [3pts]

(c) Find a k that achieves an infinite gain margin and achieves a steady state error of below 0.1 for a ramp input, $1/s^2$. [4pts]